TABLE II. Comparison of adiabatic elastic stiffness and compliance constants with isothermal values measured by Bridgman. The values in parentheses are calculated adiabatic values.

	Elastic constants of Bi Echo method d/cm²	Bridgman d/cm²
C_{11}	63.5×10^{10}	62.9×10^{10} (63.3)
C_{12}	24.7×10^{10}	35.0×10^{10} (35.56)
C_{13}	24.5×10^{10}	21.1×10^{10} (21.6)
C_{33}	38.1×10^{10}	44.0×10^{10} (44.35)
C_{44}	11.30×10^{10}	10.84×10^{10}
C_{14}	$+7.23\times10^{10}$	-4.23×10^{10}
C_{66}	19.40×10^{10}	13.87×10^{10}
S_{11}	27.8×10^{-13}	26.9×10^{-13}
S_{12}	-10.2×10^{-13}	-14.0×10^{-13}
S_{13}	-11.3×10^{-13}	-6.2×10^{-13}
S_{33}	40.8×10^{-13}	28.7×10^{-13}
S_{44}	130.7×10^{-13}	104.8×10^{-13}
S_{14}	-24.4×10^{-13}	16.0×10^{-13}
S_{66}	76.0×10^{-13}	81.2×10^{-13}

$$2\rho v_9^2=\tfrac{1}{2}(c_{11}+c_{33})+c_{44}-c_{14}+\{(\tfrac{1}{2}c_{11}-\tfrac{1}{2}c_{33}-c_{14})^2+(c_{13}+c_{44}-c_{14})^2\}^{\frac{1}{2}} \quad (14)$$

$$2\rho v_{11}^2=\tfrac{1}{2}(\tfrac{1}{2}c_{11}+\tfrac{1}{2}c_{33}+c_{44}-c_{14})-\{(\tfrac{1}{2}c_{11}-\tfrac{1}{2}c_{33}-c_{14})^2+(c_{13}+c_{44}-c_{14})^2\}^{\frac{1}{2}}, \quad (15)$$

which have been used to determine c_{13}. The error in the latter is larger than for the other five constants since the error in constants derived solely from crystals not oriented along principal axes is inherently larger, being proportional to the error in misorientation rather than to the square as for a crystal oriented along principal axes.

If one proceeds to insert the measured velocities in Eq. (2) to Eq. (15) to determine the elastic constants, one should have eight redundant checks because there are only six unknowns and 14 equations. In particular, the trace of the Christofel determinant for the X-cut crystal (i.e., $v_1^2+v_2^2+v_3^2$) should equal that for the Y-cut crystal $v_4^2+v_5^2+v_6^2$. The two observed traces are 9.580×10^{10} cm²/sec² and 9.654×10^{10} cm²/sec², respectively. Similarly, the traces of $\varphi=+90°$ and $\varphi=-90°$ crystals should be equal. In this case, the observed values are 7.974×10^{10} cm²/sec² and 7.899×10^{10} cm²/sec², respectively. Further checks of similar nature are easily made by considering sums and differences of pairs of equations such as Eq. (6) and Eq. (7).

Our actual procedure was somewhat different. The values of c_{11}, c_{33}, c_{66}, c_{44}, and c_{14} were calculated from from Eq. (2)–Eq. (11). The value of c_{11} was then slightly readjusted within the experimental error to improve the over-all agreement with all the equations. No effort was made to use a least-square procedure in view of the labor involved. The values of these constants so determined are given in Table II. The value of c_{13} is determined by solution of Eq. (12) – Eq. (15). One obtains two solutions for each pair of the four equations. The two pairs of equations have only one common root, however, and only this root will yield a positive volume compressibility. This value is also given in Table II.

It should be noted that the sign of c_{14} is positive because of the convention adopted in defining our axes. For problems involving only the propagation of sound, this convention is of no importance. However, when one desires to investigate the interaction of these sound waves with electrons, it is imperative that the sign convention adopted for designating the elastic properties be the same as that for describing the Fermi surface of the carriers. We have adopted Cady's convention because of its widespread use in the description of quartz crystals.

The values of the elastic constants reported in Table II satisfy all the stability requirements on the lattice. These conditions are easily derived by requiring the determinant of the elastic constants, corresponding to the matrix of Eq. (1), and all its principal minors, be positive in the manner described by Alers and Neighbors.[18] In calculating the elastic constants, the density of bismuth was taken to be 9.80 g/cm³ in accordance with the latest precision determination of the lattice parameters by Barrett.[19]

In addition to the measurements at room temperature, all the elastic constants except c_{13} were determined also at the boiling point of liquid N_2 and of liquid He. These values are given in Table III. Corrections for the change in length of the bismuth with temperature were made with the thermal expansion data of Erfling.[20] The determination of c_{13} as a function of temperature was not carried out owing to the high uncertainty.

DISCUSSION

In Table II, for purposes of comparison with our values, we have recorded the static values of the elastic

TABLE III. Temperature dependence of the elastic constants in units of 10^{10} d/cm².

	300°K	80°K	4.2°K
c_{11}	63.5	68.6	68.7
c_{33}	38.1	40.6	40.6
c_{44}	11.3	12.7	12.9
c_{66}	19.4	22.4	22.5
c_{14}	7.23	8.05	8.44
c_{13}	24.5	...	...

TABLE IV. Comparison of adiabatic linear and volume compressibilities.

	Echo technique cm²/d	Bridgman[a] cm²/d
K_z	18.16×10^{-13}	16.13×10^{-13}
K_t	6.28	6.59
K_v	30.7	29.31

[a] Calculated from adiabatic values.

[18] G. P. Alers and J. R. Neighbors, J. Appl. Phys. 28, 1514 (1957)

[19] C. S. Barrett, Australian J. Phys. (to be published).

[20] H. D. Erfling, Ann. Physik 34, 136 (1939).